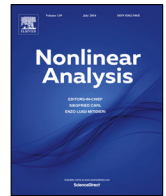




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Corrigendum

Corrigendum to “On a quasilinear elliptic problem with convection term and nonlinear boundary condition” [Nonlinear Anal. 187 (2019) 159–169]

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ABSTRACT

We correct the proof of Theorem 4.6 in “On a quasilinear elliptic problem with convection term and nonlinear boundary condition” [Nonlinear Anal. 187 (2019) 159–169].

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Since inequality (4.13) in [1] is not true in general, the proof of [Theorem 4.6](#) has to be amended. Accordingly, we need to change condition (U1) while (U2) remains the same. The assumptions read as follows.

(U1) There exist $c_1, c_2, c_3 \in \mathbb{R}_+$ such that $c_2 > c_3$ and

$$\begin{aligned} (f(x, s, \xi) - f(x, t, \xi))(s - t) &\leq c_1 |s - t|^p \quad \forall x \in \Omega, s, t \in \mathbb{R}, \xi \in \mathbb{R}^N, \\ (g(x, s) - g(x, t))(s - t) &\leq c_2 |s - t|^p - c_3 |s - t|^2 \quad \forall x \in \partial\Omega, s, t \in \mathbb{R}. \end{aligned}$$

(U2) With appropriate $\rho \in L^{r'}(\Omega)$, where $1 < r' < p^*$, and $c_4 \in \mathbb{R}_+$ one has both $\xi \mapsto f(x, s, \xi) - \rho(x)$ linear for every $(x, s) \in \Omega \times \mathbb{R}$ and

$$|f(x, s, \xi) - \rho(x)| \leq c_4 |\xi| \quad \text{in } \Omega \times \mathbb{R} \times \mathbb{R}^N.$$

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We can now formulate our uniqueness result.

Theorem 4.6. *Let (H), (U1), and (U2) be satisfied.*

(a) *If $p := 2 > q > 1$ and*

$$\max \left\{ c_1, \frac{c_2 - c_3}{\zeta} \right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} < 1 \tag{4.7}$$

then (P_μ) admits a unique weak solution for every $\mu > 0$.

(b) *If $p > q := 2$, then (P_μ) possesses only one weak solution provided*

$$\max \left\{ c_1, \frac{c_2}{\zeta} \right\} < 2^{2-p} \quad \text{and} \quad \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} < \min \left\{ \mu, \frac{c_3}{\zeta} \right\}. \tag{4.8}$$

Proof. Fix $\mu > 0$. Theorem 4.1 gives a weak solution $u_\mu \in W^{1,p}(\Omega) \cap L^\infty(\Omega)$ of (P_μ) . Suppose $v_\mu \in W^{1,p}(\Omega)$ enjoys the same property. Using (3.7) with $\varphi := u_\mu - v_\mu$ easily leads to

$$\begin{aligned} & \langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu \rangle + \mu \langle A_q(u_\mu) - A_q(v_\mu), u_\mu - v_\mu \rangle \\ & + \zeta \int_{\partial\Omega} (|u_\mu|^{p-2}u_\mu - |v_\mu|^{p-2}v_\mu)(u_\mu - v_\mu) \, d\sigma \\ & = \int_{\Omega} (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla u_\mu))(u_\mu - v_\mu) \, dx \\ & + \int_{\Omega} (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu) \, dx \\ & + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu) \, d\sigma. \end{aligned} \tag{4.9}$$

(a) Let $p := 2 > q > 1$. By monotonicity of A_q , the left-hand side in (4.9) can be estimated through

$$\begin{aligned} & \langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu \rangle + \mu \langle A_q(u_\mu) - A_q(v_\mu), u_\mu - v_\mu \rangle \\ & + \zeta \int_{\partial\Omega} (u_\mu - v_\mu)(u_\mu - v_\mu) \, d\sigma \\ & \geq \|\nabla(u_\mu - v_\mu)\|_2^2 + \zeta \|u_\mu - v_\mu\|_{2,\partial\Omega}^2 = \|u_\mu - v_\mu\|_{\zeta,2}^2, \end{aligned} \tag{4.10}$$

where $\|\cdot\|_{\zeta,2}$ denotes the equivalent norm (2.1). As regards the right-hand side, due to (U1), (U2), Hölder’s inequality, and (3.14), we have

$$\begin{aligned} & \int_{\Omega} (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla u_\mu))(u_\mu - v_\mu) \, dx \\ & + \int_{\Omega} (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu) \, dx \\ & + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu) \, d\sigma \\ & \leq c_1 \|u_\mu - v_\mu\|_2^2 + \int_{\Omega} \left(f \left(x, v_\mu, \nabla \left(\frac{1}{2}(u_\mu - v_\mu)^2 \right) \right) - \rho(x) \right) \, dx \\ & + c_2 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2 - c_3 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\ & \leq \max \left\{ c_1, \frac{c_2 - c_3}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,2}^2 + c_4 \int_{\Omega} |u_\mu - v_\mu| |\nabla(u_\mu - v_\mu)| \, dx \\ & \leq \left(\max \left\{ c_1, \frac{c_2 - c_3}{\zeta} \right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} \right) \|u_\mu - v_\mu\|_{\zeta,2}^2. \end{aligned} \tag{4.11}$$

Gathering (4.9)–(4.11) together now yields

$$\|u_\mu - v_\mu\|_{\zeta,2}^2 \leq \left(\max \left\{ c_1, \frac{c_2 - c_3}{\zeta} \right\} + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} \right) \|u_\mu - v_\mu\|_{\zeta,2}^2,$$

which implies $u_\mu = v_\mu$ because of (4.7).

(b) Let $p > q := 2$. Likewise before, the left-hand side of (4.9) becomes

$$\begin{aligned} & \langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu \rangle + \mu \langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu \rangle \\ & \quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2} u_\mu - |v_\mu|^{p-2} v_\mu \right) (u_\mu - v_\mu) \, d\sigma \\ & \geq 2^{2-p} \|\nabla(u_\mu - v_\mu)\|_p^p + \mu \|\nabla(u_\mu - v_\mu)\|_2^2 \\ & \quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2} u_\mu - |v_\mu|^{p-2} v_\mu \right) (u_\mu - v_\mu) \, d\sigma, \end{aligned} \tag{4.12}$$

while (2.2) entails

$$\int_{\partial\Omega} \left(|u_\mu|^{p-2} u_\mu - |v_\mu|^{p-2} v_\mu \right) (u_\mu - v_\mu) \, d\sigma \geq 2^{2-p} \|u_\mu - v_\mu\|_{p,\partial\Omega}^p. \tag{4.13}$$

Thus, from (4.12)–(4.13) it follows

$$\begin{aligned} & \langle A_p(u_\mu) - A_p(v_\mu), u_\mu - v_\mu \rangle + \mu \langle A_2(u_\mu) - A_2(v_\mu), u_\mu - v_\mu \rangle \\ & \quad + \zeta \int_{\partial\Omega} \left(|u_\mu|^{p-2} u_\mu - |v_\mu|^{p-2} v_\mu \right) (u_\mu - v_\mu) \, d\sigma \\ & \geq 2^{2-p} \|u_\mu - v_\mu\|_{\zeta,p}^p + \mu \|\nabla(u_\mu - v_\mu)\|_2^2. \end{aligned} \tag{4.14}$$

As in (a), by applying (U1), (U2), Hölder’s inequality, and (3.14), we have for the right-hand side of (4.9)

$$\begin{aligned} & \int_{\Omega} (f(x, u_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu) \, dx \\ & \quad + \int_{\Omega} (f(x, v_\mu, \nabla u_\mu) - f(x, v_\mu, \nabla v_\mu))(u_\mu - v_\mu) \, dx \\ & \quad + \int_{\partial\Omega} (g(x, u_\mu) - g(x, v_\mu))(u_\mu - v_\mu) \, d\sigma \\ & \leq c_1 \|u_\mu - v_\mu\|_p^p + \int_{\Omega} \left(f \left(x, v_\mu, \nabla \left(\frac{1}{2}(u_\mu - v_\mu)^2 \right) \right) - \rho(x) \right) dx \\ & \quad + c_2 \|u_\mu - v_\mu\|_{p,\partial\Omega}^p - c_3 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\ & \leq \max \left\{ c_1, \frac{c_2}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + c_4 \int_{\Omega} |u_\mu - v_\mu| |\nabla(u_\mu - v_\mu)| \, dx \\ & \quad - c_3 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2 \\ & \leq \max \left\{ c_1, \frac{c_2}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} \|u_\mu - v_\mu\|_{\zeta,2}^2 - c_3 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2. \end{aligned} \tag{4.15}$$

Combining (4.9) with (4.14)–(4.15) yields

$$\begin{aligned} & 2^{2-p} \|u_\mu - v_\mu\|_{\zeta,p}^p + \mu \|\nabla(u_\mu - v_\mu)\|_2^2 \\ & \leq \max \left\{ c_1, \frac{c_2}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} \|u_\mu - v_\mu\|_{\zeta,2}^2 - c_3 \|u_\mu - v_\mu\|_{2,\partial\Omega}^2, \end{aligned}$$

which directly leads to

$$\begin{aligned} & 2^{2-p} \|u_\mu - v_\mu\|_{\zeta,p}^p + \min \left\{ \mu, \frac{c_3}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,2}^2 \\ & \leq \max \left\{ c_1, \frac{c_2}{\zeta} \right\} \|u_\mu - v_\mu\|_{\zeta,p}^p + \frac{c_4}{\sqrt{\lambda_{1,2,\beta}}} \|u_\mu - v_\mu\|_{\zeta,2}^2. \end{aligned}$$

Therefore, if (4.8) is satisfied, then $u_\mu = v_\mu$.

References

- [1] S.A. Marano, P. Winkert, On a quasilinear elliptic problem with convection term and nonlinear boundary condition, *Nonlinear Anal.* 187 (2019) 159–169.